Written exam in the study of Economics

LINEAR MODELS

Monday, June 25, 2012.

(3 hours written exam. All ususal aids allowed (i.e. books, notes etc.), but it is neither allowed bringing any electronic calculator nor using any other electronic equipment. Open book exam.)

The language of the exam is English.

KØBENHAVNS UNIVERSITETS ØKONOMISKE INSTITUT

2012Su-2LM ex

Examination in Linear Models

Monday, June 25, 2012.

This is a 3 hour examination (2 pages with a total of 4 exercises).

It is allowed using textbooks, notes and personal notes. It is strictly prohibited using calculators or cas tools.

Problem 1. In $\mathbf{R}^n$, $n > 4$, the 3 vectors u_1, u_2 and u_3 form a basis for a subspace U. Moreover, the vectors u_4 and u_5 are given by $u_4 = u_1 + u_2 + u_3$ and $u_5 = u_1 + u_2 - u_3$.

(1) Determine the coordinates of $u_4 + u_5$ with respect to the basis u_1, u_2, u_3 in U.

(2) Let the linear mapping $S : U \to U$ be given by $Su_1 = u_2 + u_3$, $S(u_1 + u_2) = u_4$ and $Su_3 = u_2$. Determine the matrix corresponding to S with respect to the basis u_1, u_2, u_3 in U.

(3) Show that S is bijective, and find $S^{-1}u_2$.

(4) Show that u_1, u_2, u_4 also form a basis for U.

(5) Find the matrix corresponding to S with respect to the basis u_1, u_2, u_4 for U.

Problem 2.

The $3 \times 3-$matrix A has the eigenvalues $1, 2$ and 3, with corresponding eigenvectors $(-2, 1, 1), (0, -1, 1)$ and $(1, 1, 1)$.

(1) Determine one of the possible versions of the matrix A.

(2) Find the matrix $\ln(A)$ (where ln denotes the natural logarithm).

(3) Show that A is invertible and that $\ln(A)$ is not invertible.

(4) Find the eigenvalues for A^{-n}, where n is a natural number.

Problem 3.

(1) Calculate the integral $\int \sin(x)\sin(2x)\cos(3x)dx$.

(2) Solve the equation $6z^2 - 12z + 12 = 0$.

Problem 4.

We consider the series

$$\sum_{n=0}^{\infty}(1-x^2)^n.$$

(1) Find the values of x for which the series is convergent.

(2) Find an expression for the function $f(x) = \sum_{n=0}^{\infty}(1-x^2)^n$.

(3) Determine the range, $R(f)$, for the function f.

(4) Find an expression for the function $g(x) = \sum_{n=0}^{\infty}(n+1)(1-x^2)^n$.